Balance of humanoid robots by proper foot placement

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1 Motivation and related work

When humanoid robots are going to be used in society, they should be capable to maintain their balance. Balance of bipedal robots is defined as the state of the robot in which it is not fallen. We say that a robot is/remains in balance as long as it did not fall/is not going to fall. Balance of bipedal robots has been analyzed using numerous approaches. These approaches can rigorously be divided into two groups.

First, there are approaches that consider tipping of the stance foot as criterion for balance. In these approaches, balance is commonly evaluated using the zero moment point [5] or foot rotation indicator [1]. These points can be actively controlled to stay in the support polygon of the feet, so that the biped remains balanced. Indeed, as long as the stance foot of the biped does not tip, the biped stays fully actuated which guarantees that it does not fall. However, this is rather conservative [3], because to maintain the balance it is not necessary that the stance foot does not tip. Humans are the perfect counter example, as we tip our stance feet during walking.

The second group of criteria considers foot placement for balance. Indeed, a humanoid robot maintains its balance if it knows where to put its swing foot in order not to fall. So knowing where to step is crucial for the balance of a biped. Strategies that compute where to step in order to keep the balance include, but are not limited to: capture points [2] and the foot placement estimator [7].

These strategies commonly use conservation of energy to determine proper foot placement. With proper foot placement we mean the point on the floor where the biped can step such that, after the step, its center of mass moves precisely above its stance foot exactly when all the energy is converted into solely potential energy. So the biped stops in single support in this unstable equilibrium point, which we define as the balanced configuration. The aforementioned strategies use a very simple (linear) inverted pendulum model to determine where to step. These models describe some key aspects of bipedal walking quiet accurately so that they can be used for basic analysis and control of humanoid robots. However, not all dynamic aspects of a complex humanoid robot are captured in an inverted pendulum model. Thus, since approximately a decade, interest has grown in full body analysis and control, for example in the area of angular momentum control. We believe that also balance control by proper foot placement can benefit from more complex models.

Namely, the commonly used methods rely on knowledge of the kinetic and potential energy of the inverted pendulum model. This energy is of course dependent on the configuration of the bipeds legs, arms and torso. So it is arguable if the actual energy contained in a complex biped with non-massless legs, arms and torso can accurately be estimated using an inverted pendulum model. For example, when the biped has to recover from a significant push, which is a common source of unbalance, it has to move its swing leg in a short time to a desired step location. This short reaction time automatically results in high velocity, aggressive movements that influence the dynamics of the entire biped. Another example is the dynamics that occur at the moment of support change. The linear inverted pendulum model does not include discontinuous impact dynamics. In our opinion, the instantaneous swapping of the stance and swing leg without a double support phase, does influence the energy contained in the system. The collision of the rigid swing foot with the ground may result in discontinuous jumps in the velocities and thus in the kinetic energy of the system. Therefore, we believe that more complex models that take into account the full dynamics of the biped including ground impact are important to compute proper foot placement.

2 Foot placement for complex bipeds

Recently, we developed an algorithm that computes proper foot placement for complex bipeds with an arbitrary number of degrees of freedom [4]. We call this algorithm the foot placement indicator (FPI), which is an extension of the foot placement estimator [7]. The algorithm works as follows. First, we model an N-link biped as an impulsive system [6]:

$$\Sigma = \begin{cases} D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u, & \mathscr{S} \neq 0, \quad \text{(1a)} \\ \dot{q}_+ = \Delta(q_-)\dot{q}_-, & \mathscr{S} = 0, \quad \text{(1b)} \end{cases}$$

where $q \in \mathbb{T}^N$ is the state vector, $D \in \mathbb{R}^{N\times N}$ is the symmetric positive definite inertia matrix, $C\dot{q} \in \mathbb{R}^N$ is the vector containing Coriolis and centrifugal terms, $G \in \mathbb{R}^N$ is the gravity

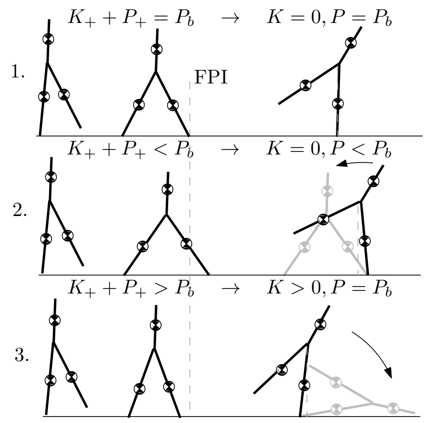


Figure 1: Proper foot placement on FPI for balance

vector and $u \in \mathbb{R}^N$ is the input vector. The impact surface is given by $\mathscr{S}$ and if a state just before impact q_- hits this surface, it is discontinuously mapped to the state just after impact q_+ by the impact matrix $\Delta \in \mathbb{R}^{N \times N}$. Now, in this modeling framework, the FPI algorithm uses conservation of energy during a step, see Figure 1. Unlike (linear) inverted pendulum models, this algorithm takes into account the potential and kinetic energy of all links of the biped and the impulsive, discontinuous impact dynamics to determine proper foot placement. Using model (1) we assume that an impact is going to occur in the next time instant and that after this impact the current internal configuration of the biped is locked so that energy is conserved. Using this locked configuration, we can find an expression for the potential energy in the balanced configuration. We equate this potential energy to the expression for the total energy and solve for the joint angles of the system in:

$$K_+(q_-, \Delta(q_-)\dot{q}_-) + P_+(q_-) = P_b(q_-), \tag{2}$$

where K_+ and P_+ are respectively the kinetic and potential energy just after the impact and P_b is the energy of the balanced configuration. Solving (2) for the joint angles q_- gives the proper foot placement configuration, because if an impact occurs in this configuration, the energy in the system after the impact is equal to the energy in the balanced configuration and, hence, the system evolves to the balanced configuration, see also Figure 1. The solution of (2) can be used as reference configuration q_{FPI} in a feedback tracking controller to keep the robot balanced.

The main limitation of this method is the fact that the solution of (2) is in general not unique for bipeds with more than two degrees of freedom. Namely, there might be more pre-impact configurations such that the biped evolves to a balanced standing configuration. Multiple options exist for finding a solution for bipeds with more degrees of freedom. The most elegant one is to use virtual holonomic constraints [6] to relate certain joints angles to each other. In essence, we constrain the motion of the high order model to a lower one, but still take into account the kinetic and potential energies of all links in the system. Alternatively, we could use nonlinear optimization methods.

3 Simulation example on planer five-link biped

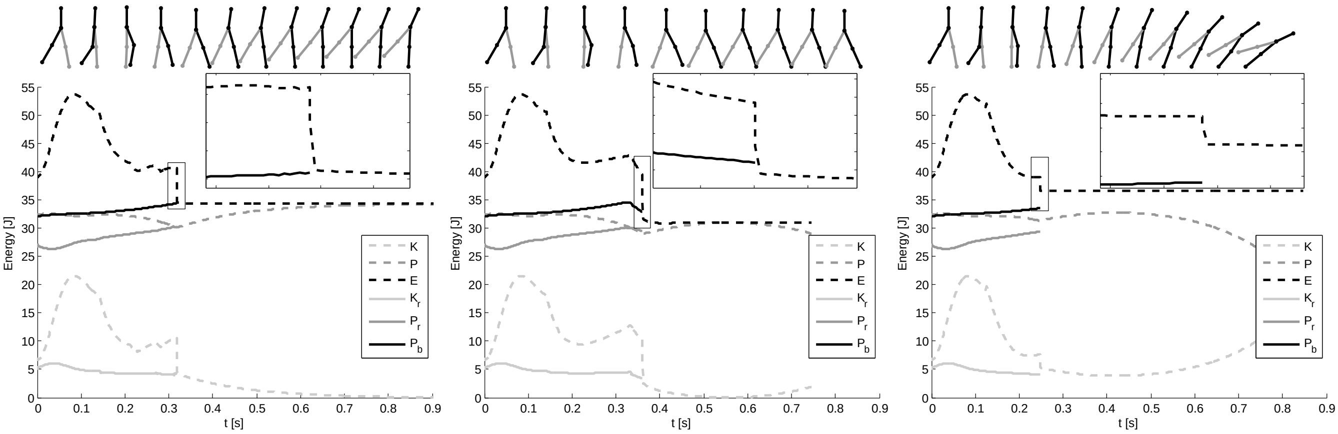
We show the three different scenarios as shown in Figure 1 using numerical simulations of a planar five-link biped. The results of these simulations are depicted in Figure 2. In the three simulations we compute the FPI point and show the results for a biped stepping exactly onto the FPI point (scenario 1, Figure 2(a)), after the FPI point (scenario 2, Figure 2(b)) and before the FPI point (scenario 3, Figure 2(c)). These plots show relevant energies: the actual kinetic energy K, potential energy P and total energy E and the estimated post-impact kinetic energy K_r, post-impact potential energy P_r and final potential energy in the balanced standing configuration P_b. In each simulation, the robot starts with the same initial configuration and velocity, but we intentionally adjusted the accuracy of the joint PD tracking controllers in two simulations to show the differences between the three scenarios. The FPI algorithm computes at each time instant before the impact the FPI configuration q_{FPI} from (2). Feedback controllers bring the actual q to the FPI configuration.

We can observe in each simulation that before the impact, the biped falls forward, so the kinetic energy K increases whereas the potential energy P decreases. At the impact, a clearly visible drop in kinetic energy can be noticed, whereas the potential energy is constant over the impact. This also results in a steep drop in the total energy E at the impact. After the impact we lock the joints, so that the total energy E in the system remains constant. The biped continues its motion forward and converts the kinetic energy into potential energy. The differences between the results are:

a) the FPI method accurately estimates the post-impact kinetic energy K_r, post-impact potential energy P_r and balanced standing configuration potential energy P_b, since these coincide with the actual kinetic energy K, potential energy P and total energy E after the impact. When the joints lock, the total energy in the system remains constant, so the relation $K_r + P_r = P_b$ holds. Indeed, we observe that the biped converts all the kinetic energy into potential energy and stops at the balanced standing configuration.

b) the FPI method overestimates the post-impact kinetic energy K_r, post-impact potential energy P_r and balanced standing configuration potential energy P_b. When the joints lock, the total energy remains constant, but is smaller than the required balanced standing configuration potential energy: $K_r + P_r = P_b > E_r$, where E_r is the actual total energy in the system after the impact. The biped converts all the kinetic energy into potential energy before it reaches the balanced standing configuration and falls back.

c) the FPI method underestimates the post-impact kinetic energy K_r, post-impact potential energy P_r and balanced standing configuration potential energy P_b. When the joints lock, the total energy remains constant, but is larger than the required balanced stand-



(a) Relevant energies in scenario 1. (b) Relevant energies in scenario 2. (c) Relevant energies in scenario 3.

Figure 2: Simulation results for the three different scenarios. Figures show relevant energies: actual kinetic energy K, actual potential energy P, actual total energy E, estimated post-impact kinetic energy K_r, estimated post-impact potential energy P_r and estimated balanced configuration potential energy P_b. The inset is a magnification of the marked area. The snapshots above the plots show the corresponding motion of the biped.

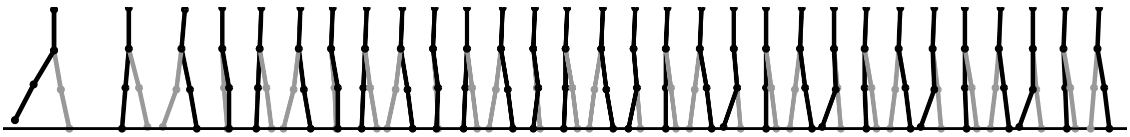


Figure 3: Snapshots of a balanced walking gait by continuously stepping before the FPI point. Stability of the corresponding limit cycle has been verified by Poincaré's theorem.

ing configuration potential energy: $K_r + P_r = P_b < E_r$. The biped reaches the balanced standing configuration with nonzero kinetic energy and falls.

These three simulations clearly show that we can accurately compute the foot placement location for planar bipeds with point feet. If we apply proper control, the biped can step exactly onto the desired foot location and it remains balanced.

In another simulation, we continue scenario 2 and let the FPI algorithm compute step locations for subsequent steps. This eventually results in balanced walking as can be seen in Figure 3. Stability of the resulting limit cycle has been verified through the eigenvalues of the Poincaré return map.

4 Conclusion

In this paper we explained balance of bipedal robots using the notion of the foot placement indicator (FPI). The FPI method can accurately compute where a biped needs to step in order to evolve to a balanced standing configuration, i.e. the configuration in which the biped is in equilibrium and its center of mass lies exactly above its stance foot. The method is based on energy conservation in the system, properly taking into account energy losses during the impact. We showed applicability of the algorithm in simulations of a five-link planar biped.

References

[1] A. Goswami. Foot rotation indicator (FRI) point: A new gait planning tool to evaluate postural stability of biped robots. In *Proceedings of the IEEE International Conference on Robotics and Automation*, pages 47–52, 1999.

[2] J. Pratt, J. Carff, S. Drakunov, and A. Goswami. Capture point: A step toward humanoid push recovery. In *6th IEEE-RAS International Conference on Humanoid Robots*, pages 200–207, 2006.

[3] P.W.M. van Zutven, D. Kostić, and H. Nijmeijer. On the stability of bipedal walking. *LNAI series: Simulation, Modeling, and Programming for Autonomous Robots*, pages 521–532, 2010.

[4] P.W.M. van Zutven, D. Kostić, and H. Nijmeijer. Foot placement for planar bipeds with point feet. In *Proceedings of the IEEE International Conference on Robotics and Automation*, 2012 (in press).

[5] M. Vukobratović and B. Borovac. Zero-moment point-thirty five years of its life. *International Journal of Humanoid Robotics*, pages 157–173, 2004.

[6] E.R. Westervelt, J.W. Grizzle, C. Chevallereau, J.H. Choi, and B. Morris. *Feedback control of dynamic bipedal robot locomotion*. New York: CRC Press, 2007.

[7] D.L. Wight, E.G. Kubica, and D.W. L. Wang. Introduction of the foot placement estimator: A dynamic measure of balance for bipedal robotics. *Journal of Computational and Nonlinear Dynamics*, pages 1–9, 2008.