A gravitational impulse model predicts optimality of push-off impulse during gait

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Keywords: Collision, Impulse, Push-off, Heel strike, Gravity, Step-to-step transition

1 Introduction

Simplest walking model could successfully describe the basic mechanics of human walking, while the prediction of collision dynamics and related energetics were restricted due to the non-realistic, instantaneous collision assumption. Specifically, to apply a finite impulse to induce a finite momentum change during the step-to-step transition, the simplest walking model assumes infinitely large impulsive forces and negligible effect of the finite gravity force. However, during the real human walking, the collision occurs over a finite duration of time around 0.1 to 0.2 sec following finite impulsive ground reaction forces (GRFs). Therefore, the order of magnitude of the gravity force is compatible to that of GRF. In this study, we propose a new collision model that introduces the contribution of the gravitational impulse to the momentum change of the CoM during a step-to-step transition to supplement the limitations of collision prediction of the previous model.

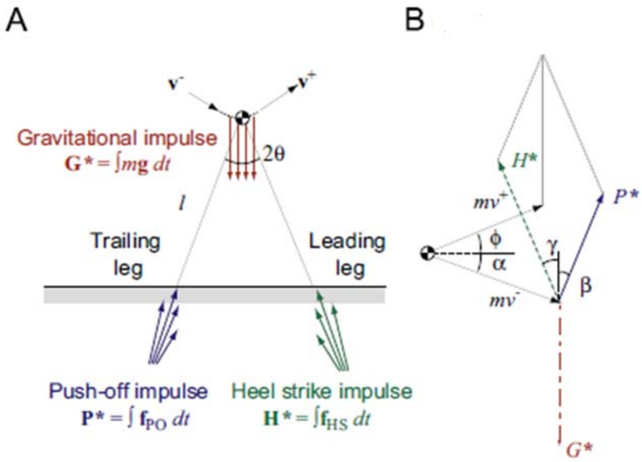
2 Methods

We expanded the equations of collisional work by *n*-simultaneous impulses and quantitatively analyzed the collision mechanics and energetic [1]. To validate the model, we measured the GRFs of six subjects' over-ground walking at five different gait speeds and calculated the collision impulses and mechanical work. We also examined whether the push-off propulsion was performed in an energetically optimal manner.

Specifically, when the GRFs and the gravity are simultaneously applied during the double support phase, the work done to the CoM by each impulse is written as the form of

$$\begin{cases} W_{push} = \dfrac{P^*}{2m}\left\{P^* + 2mv^- \sin(\beta-\alpha) + H^*_{hp}\cos(\beta+\gamma) - G^*_{gp}\cos\beta\right\} \\ W_{gravity} = \dfrac{G^*}{2m}\left\{G^* + 2mv^- \sin\alpha - P^*_{pg}\cos\beta - H^*_{hg}\cos\gamma\right\} \\ W_{heel} = \dfrac{H^*}{2m}\left\{H^* - 2mv^- \sin(\alpha+\gamma) + P^*_{ph}\cos(\beta+\gamma) - G^*_{gh}\cos\gamma\right\} \end{cases}$$

where α, β, and γ are the directions of pre-collision velocity of the COM push-off, and heel strike impulses, respectively (Fig. 1B). The magnitude of each vector component is denoted by a non-bold letter. The weighted impulse of $\mathbf{I}^* = \int_{t^-}^{t^+} \mathbf{f} dt$ is defined as $\mathbf{I}^*_{ik} \equiv 2s_{ik}\mathbf{I}^*$, with a collision overlap parameter s_{ik} of impulse $\mathbf{I}_i$ with respect to $\mathbf{I}_k$ defined as $s_{ik} \equiv \int_0^1 \rho_i d\rho_k$ where $\rho_k(t) \equiv \left(\mathbf{I}_k(t)\cdot\mathbf{I}^*_k\right) / \left|\mathbf{I}^*_k\right|^2$ [1,2].



[Figure. 1] Momentum-impulse diagram of the CoM during step-to-step transition. Note that the gravitational impulse $\mathbf{G}^*$ also contributes to the momentum change. Figures were adapted from [2].

To quantitatively compare the contribution of the gravitational impulse to the condition of optimal gait, we simplified the model using the massless leg ($\alpha = \beta = \gamma = \phi = \theta$, Fig. 1) and truly simultaneous collision (s_{ik} = 0.5, (Ruina et al., 2005)) assumptions. By substituting these constraints into Eqs. (1) and (2b), W_{ss} and W_{push} are obtained as quadratic forms of push-off impulse P^* as follows:

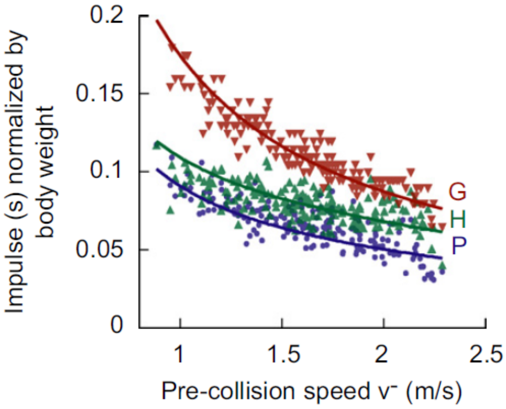
$$\begin{cases} W_{push} = \dfrac{P^*}{2m}\sin^2 2\theta\left(P^* + \dfrac{mv^-}{\tan 2\theta} - \dfrac{G^*}{2\cos\theta}\right) \\ W_{gravity} = \dfrac{G^*}{2m}\sin^2\theta\{G^* + 2(mv^-\sin\theta - P^*\cos\theta)\} \\ W_{heel} = -\dfrac{1}{2}v^-\sin 2\theta\{mv^-\sin 2\theta - P^*\cos 2\theta + G^*\cos\theta\} \\ W_{ss} = -(W_{push} + W_{heel}) \\ \quad = \dfrac{-1}{2}\left\{P^* - \left(mv^-\tan\theta + \dfrac{G^*}{2\cos\theta}\right)\right\}\left(P^* + \dfrac{mv^-}{\tan\theta}\right) \end{cases}$$

Suppose the possibility of negative work on the CoM during the single stance phase is discounted [3], then the optimal push-off is obtained as $P^*_{opt} = mv^-\tan\theta + G^*/2\cos\theta$.

To validate the newly proposed gravitational impulse model, we measured the GRF, impulse and mechanical work done on the CoM for six healthy young subjects (mean age 22.8 ± 1.2 years). All subjects signed the informed consent form approved by the Institutional Review Board of KAIST prior to the test. Subjects walked on a 10-m long and 1-m wide walkway at five different walking speeds. Details of the test and analysis were described in [2].

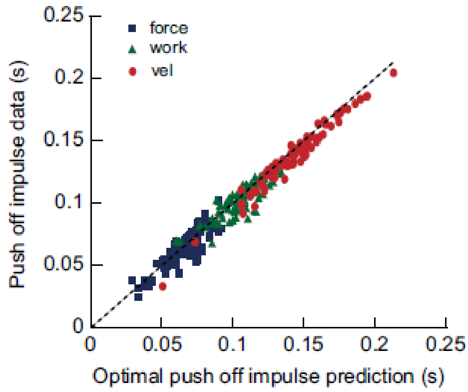
3 Results

Empirical data of impulse and mechanical work support the new collision model. Specifically, the decreasing trend of the collision impulse during the double support phase was well matched by the optimal gait prediction (Fig. 2) implying the step-to-step transition occurred in an energetically optimal manner. The vector sum of collision impulses including the gravitational impulse well represent the momentum change of the COM during the step-to-step transition [2], implying significant contribution of the gravity to the momentum change which were ignored in the previous no-gravity model.



[Figure. 2] Experimental data and model predictions (solid lines) of optimal push-off (filled circles), heel-strike (triangles), and gravitational impulse (reverse-triangles) during double support phase. Figures were adapted from [2].

To examine the robustness of the energetic optimality of the push-off during walking for different step-to-step transition, we also examined and the push-off and heel strike work for three differently defined step-to-step transitions based on the force, work, and velocity [4]. The results showed that the push-off work on the CoM was well matched by the optimal impulse predictions (R2 > 0.7) for all three definitions of the step-to-step transition. The results imply that the balance of push-off and heel strike energy is a consistent property of gait dynamics, and an inherited oscillatory behavior of the CoM, would be possibly inherited by spring-like leg mechanics.



[Figure. 3] Push-off impulse data versus the predicted optimal push-off that was computed from equations that minimize the CoM work in the model for the three definitions of the step-to-step transitions based on the the force (squares), work (triangles), and velocity (circles). Figures were adapted from [4].

4 Conclusions

The new model predicted a moderate change in the collision mechanics with gait speed, which seems to be physiologically achievable. The gravitational collision model enables us to better understand collision dynamics during a step-to-step transition.

5 Open Questions

Other than the steady state gait trials shown in this study, we wonder whether the proposed model could predict the push-off impulse during transient gait trials such as accelerated and/or decelerated walking, and its energetic optimality.

Acknowledgement

This work was supported from UTRC at KAIST, originally funded by DAPA, ADD.

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