Design of stable walking gaits for biped robots with several underactuated degrees of freedom

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1 Motivation

Dynamic walking gaits are theoretically reflected by the periodic solutions exhibited by nonlinear impulsive dynamical systems [1]. In practice, such solutions, known as limit cycles, can exist either as a natural response of the system, e.g. passive walkers, or by the use of feedback control. Analytically, finding these solutions is certainly challenging, and the complexity of the problem increases considerably if a mixture in between passive and actuated joints are considered, i.e. underactuated robots.

2 State of the Art

To approach this problem analytically, recent advances in nonlinear control have proposed to encode virtual holonomic constraints (VHC) by an special design of feedback controllers [2, 3]. VHC are defined as a parametrization of the generalized coordinates. This procedure is based on basic fundamental mathematical principles, that allow a tractable analytical and practical approach for trajectory planning and control design, allowing to achieve stable limit cycle motions.

The underlying concept of this approach is the choice of constraint functions, which drawback is the non-existence of a standardized procedure. Generally, this is left to the intuition of the designer, with a few hints on how to handle certain parametric polynomials. The validity of these polynomials is upon existence of a periodic solution fulfilling certain optimality conditions. However, such procedures are manageable for underactuation degree one cases, i.e. robots with passive ankles, but almost intractable for higher cases.

3 Contribution

Our main technical objective is to show a procedure to define VHC for systems with underactuation degree higher than one. To this end, we consider a classical example of a three-link planar biped walker with two passive links.

The model shown in Fig. 1 represents a planar biped walker with two symmetric legs and upper torso [4]. It is actuated in between the legs, while the use of torsional springs is considered for keeping the torso upright. These design considerations lead to a system of underactuation degree two. The task consists of designing symmetric periodic gaits in flat ground, which will allow us to present a novel systematic gait synthesis technique.

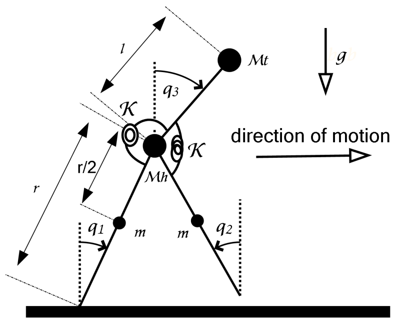

Figure 1: Schematics of the biped in the sagittal plane and level ground.

Our gait synthesis approach consists of finding a set of time independent geometric functions, which uniquely describe the instantaneous postures of the robot along the gait. This implies that rather than searching for the time evolution of the generalized coordinates, we are interested on an alternative state-dependent parametric representation of the motion. One example is

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \theta_\star(t) \\ \phi_2(\theta_\star(t)) \\ \phi_3(\theta_\star(t)) \end{bmatrix}, \qquad (1)$$

where the angle of the stance leg is used as a new parameterizing variable, with $\phi_2(\cdot)$ and $\phi_3(\cdot)$ being the functions mentioned above[1], and defined for the interval $\theta_\star \in [\Theta_b, \Theta_e]^2$. With this scheme, the explicit dependence of time disappears, and θ becomes the new trajectory generator.

This approach proposes the following:

[1]In the context of control theory, these functions are known as virtual holonomic constraints, see e.g. [5].

[2]The evolution of the stance leg q_1 varies monotonically from its initial to its final configuration.

(a) A mathematical manipulation of the system dynamics, in terms of (1), states that the constraint function ϕ_3, describing the torso's trajectory, explicitly satisfies a differential equation of the form:

$$\phi_3''' = f(\theta, \phi_2, \phi_3, \phi_2', \phi_3', \phi_2'', \phi_3'', \phi_2''') \,. \qquad (2)$$

(b) The motion of the swing leg ϕ_2 can be defined by an arbitrary function, e.g. Bézier polynomial, that encodes a desired characteristic.

(c) Defining:

$$\chi_0 = \left[K, \theta^+, \phi_2^+, \phi_3^+, \dot{\theta}^+, \dot{\phi}_2^+, \dot{\phi}_3^+\right] \,. \qquad (3)$$

as our vector of unknown parameters, a walking gait is defined by a periodic solution of (2).

One gait found by this procedure has the solution's vector (3) given by

$$\begin{aligned}\chi_\star \approx & [23.9770, -0.2503, 0.2503, 0.3131, 1.2512, \\ & 0.4956, -0.2695] \,.\end{aligned} \qquad (4)$$

The evolution of q_1 is within the step interval $[-0.2503, 0.2503]$, and the period is $T_e = 0.4878$ sec. The numerical value of energy calculated for this gait is 21,16 W/m. The constraint functions $\phi_2(\theta)$, $\phi_3(\theta)$, and the control signal $u(\theta)$ are shown in Fig. 2. The

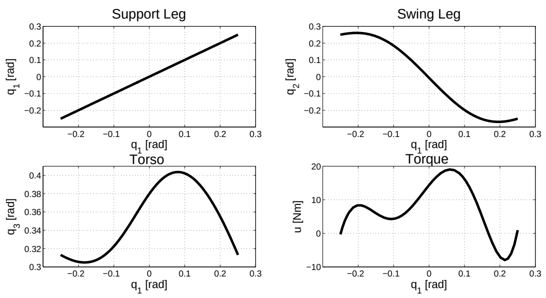

Figure 2: Trajectories as functions of q_1 with initial conditions (4). The motion goes from left to right.

gait is stabilized by feedback control, and applying an extension to the technique proposed in [6] to the case of systems with several passive links.

4 Discussion Outline

As it might have been noticed, the complexity of the problem is significantly reduced, allowing to apply similar methodologies as the very well known gait search for passive walkers, with (2) instead of the original dynamical model. The iterative procedure to find the gait allows to consider energy performance as part of the cost function.

The interesting outcomes of this procedure are a) a step-by-step procedure for planning gait cycles, and b) an exemplified approach for control design of mechanical systems presenting several passive degrees of freedom. However, there is a big room of improvement, since it does not exist standardized concepts reflecting the meaning of optimal gaits. Therefore, we are still restricted to state of the art concepts regarding the characterization of gaits. Nevertheless, our procedure allows to make the complex problem of gait synthesis mathematically tractable.

5 Format

Poster Presentation

6 Keywords

Limit cycle walking

References

[1] D.G.E. Hobbelen and M. Wisse, "Limit Cycle Walking," Humanoid Robots – Human-like Machines, I-Tech Education and Publishing, 2007, Vienna, ed. Matthias Hackel .

[2] J.W. Grizzle, G. Abba and F. Plestan, "Asymptotically Stable Walking for Biped Robots: Analysis via Systems with Impulse Effects," IEEE Transactions on Automatic Control, 2001, Volume: 46, Number. 1.

[3] L.B. Freidovich and A.S. Shiriaev, "Transverse Linearization for Mechanical Systems with Passive Links, Impulse Effects, and Friction Forces," Proceedings of Joint 48th IEEE Conference on Decision and Control, and 28th Chinese Control Conference, Shanghai, P.R. China.

[4] M. Gomes and A. Ruina, "Walking model with no energy cost," Phys. Rev. E, American Physical Society, 2011, Volume: 83, Number. 3.

[5] A.S. Shiriaev, J.W. Perram and C. Canudas de Wit, "Constructive Tool for Orbital Stabilization of Underactuated Nonlinear Systems: Virtual Constraints Approach," IEEE Trans. on Automatic Control, 2005, Volume: 50, Number. 8.

[6] A.S. Shiriaev and L.B. Freidovich, "Transverse linearization for impulsive mechanical systems with one passive link," IEEE Trans. on Automatic Control, 2009, Volume: 54, Number. 12.