

# Comparing Performance of Running Robots: Gravity, Size, Speed and Stride as Metrics for Dynamic Performance

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To compare legged robotic systems at various sizes, speeds, even gravitational environments, we need some dynamic standard of measure that is easy to understand and universally applicable. This will allow us to usefully relate robot performance to something familiar to our experience in dynamic equivalent human terms.

We will look to the familiar example of a simple pendulum as our dynamic standard of motion.

We will derive dimensionless velocity, dimensionless stride and dimensionless frequency from this dynamic reference.

Animals, people, robots of all different sizes and speeds, even operating in reduced gravity, can all be compared in this useful and intuitive manner.

Relationships and insights, some familiar, some surprising, will come from this dimensionless mapping of dynamic performance.

## State of the Art

If my robot is four times the size of yours, should it travel four times as fast to be somehow dynamically similar? Twice as fast? If these robots were the size of a human – would they be as fast as you or I? Half as fast? Twice as fast? How can we know the answers to these questions? Simply comparing speeds directly doesn't tell us these answers.

Animals and robots have recently been described according to relative speed – that is actual speed divided by leg lengths or body lengths. [1]

Relative speed, commonly defined as body lengths or leg lengths per second, can be a misleading metric to compare the dynamic performance of running robots of different sizes possibly operating in different gravitational environments.

High relative velocities tend to indicate small size, rather than true dynamic performance.

Gravity is not included in the definition.

Velocity increases only as the square root of the linear dimension ratio.

## Dimensionless Velocity is Dynamically Consistent

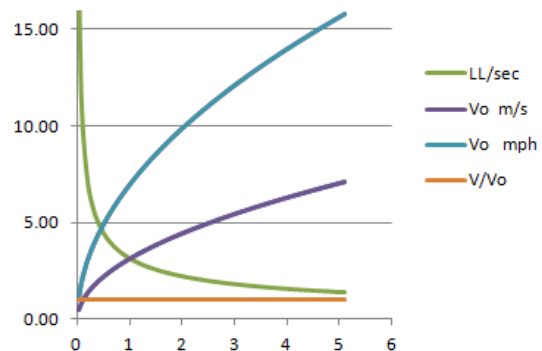


Figure 1. Dimensionless velocity for standard simple pendulums 0.025 to 5.1m is dynamically consistent 1.0 for all cases. Relative velocity in Leg Lengths per second exceeds 15 at very small sizes, and actual velocity in miles per hour exceeds 15 for very large sizes.

In the above graph, standard reference simple pendulums, varying in length from 0.025 to 5.1 meters, are released from rest at the standard sixty degrees initial angle from the vertical. As they pass through the vertical at midswing, they achieve their maximum velocity. This is the reference velocity  $V_0$ .

Dimensionless velocity  $V/V_0$  is 1.0 by definition for all of these pendulums. The dimensionless velocity would be the same for robots whose legs are the same lengths if their forward speeds correspond accordingly.

The relative velocity measure, speed divided by leg length, exceeds 15 at very small sizes, while the actual velocity in miles per hour exceeds 15 at the much larger sizes.

These measurements are heavily influenced by size, but the dimensionless speed measure is not. The dimensionless velocity is 1.0 for all of them, showing that they are all dynamically equivalent. In familiar human terms, for dynamically consistent reference, they are all the dynamic equivalent of a standard human running at 7mph.

The only speed measure that is dynamically consistent is the dimensionless velocity.

### Dimensionless Velocity

For a robot whose legs are as long as the pendulum, we can say the robot is moving just as fast, or twice as fast, or half as fast – as the reference velocity we could measure from such a pendulum.

Only size and gravity affect the speed of a simple pendulum, not the mass itself. So the reference velocity is easy to obtain for a robot of any leg size operating in any gravity.

By releasing the pendulum from rest at sixty degrees from the vertical, we have the needed reference speed  $V_0$ , as the speed obtained when the gravitational potential energy of half the height of the effective Limb is completely converted to kinetic energy of forward motion.

Dividing the robot’s actual forward velocity by this dynamic reference velocity gives is a ratio of speeds. Since we are dividing speed by speed, there are no units to this ratio of comparison. We call the result the Dimensionless Velocity.

$$\hat{V} = V / V_0$$

All simple pendulums released from this initial angle will therefore achieve a dimensionless velocity value of 1.0 at midswing as the pendulum swings its fastest as it passes through the vertical, when released as described, regardless of pendulum size or differing values of gravitational acceleration.

### Dimensionless Stride

Further, the Stride Ratio, or Dimensionless Stride, is defined as the Stride Length of a legged robot – from footfall to footfall for the same leg - divided by the height of the effective Limb.

$$\hat{L} = L / L_0$$

Notice that when we run, we naturally take strides much longer than the short strides we take when we are walking. We also take strides more frequently.

### Dimensionless Frequency

We can easily recognize that forward speed divided by stride length gives the step frequency for a legged system.

We also define Dimensionless Frequency as the Dimensionless Stride divided by the Dimensionless Speed:

$$\hat{F} = \hat{L} / \hat{V}$$

In a familiar reference example, the dimensionless frequency of 0.25 corresponds to a comfortable, efficient walking speed.

### Alexander Equation for Running Animals

We recognize the utility of the R. McNeill Alexander data and his power law approach for comparing the performance of many legged animals. [2]

Starting from the Alexander power law equation [SI units] we have

$$v = 0.25 \times \text{Gravity}^{0.5} \times \text{StrideLength}^{1.67} \times \text{Limb Height}^{-1.17}$$

Recalling that dimensionless stride is stride length divided by effective limb height, and rearranging and expressing in terms of Dimensionless Stride as the primary variable, we find:

$$\text{Dimensionless Velocity} = 0.25 \text{ Dimensionless Stride}^{1.66}$$

$$\text{Dimensionless Frequency} = 0.25 \text{ Dimensionless Stride}^{0.66}$$

In our preferred plot, dimensionless velocity, and dimensionless frequency are plotted with respect to dimensionless stride as the primary variable.

When plotted in this way, the interrelations between the variables become simple power law equations, easy to remember and understand.

Intersections of the curves will provide handy reference points with dynamic significance that can easily be related to human experience.

In the expanded abstract we will chart these values for robots, people and animals, and plot them in this three axis dimensionless space to show some familiar relationships, some new insights, and some surprising results that will stimulate further discussion as we explore and compare in a dynamically consistent way how fast we are really going for every size and gravity.

With this convenient dimensionless perspective, and with new materials, manufacturing methods and emerging dynamic robot architectures, we may be just beginning to explore how fast the legged systems we can now envision and build can really go.

### References :

[1] Adaptive, Fast, Dynamic Walking Robot "RunBot"  
<http://www.manoonpong.com/Runbot.html>

[2] University of Sheffield, Sorby Geology Group, Dinosaur Speed Calculator  
<http://www.sorbygeology.group.shef.ac.uk/DINOC01/dinocal1.html>

