Optimizing robust trajectories for legged locomotion on rough terrain

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Motivation

Not all limit cycles are created equal. Some nominal limit cycles are more energy efficient; others are more naturally robust. Here we consider a robust control formulation of walking on terrain; we would like to design a nominal limit cycle that still provides a notion of stability for every possible terrain height given by a bounded distribution.

State of the Art

By simulating the SLIP model from step to step, Ernst and Geyer find the open loop deadbeat controller which perfectly rejects terrain disturbance [1]. This is a clear demonstration that the choice of nominal trajectory can have dramatic impact on stability. We attempt to use this result as a benchmark, and would like it to emerge out of a more general formulation.

Trajectory optimization has become a standard tool in dynamic walking. Mombaur designs the self-stable open loop orbit by minimizing the spectral radius of the Poincaré map; since the spectral radius is not a differentiable function, it is optimized through direct search instead of using gradient information [4]. Morimoto and Atkeson proposed a minimax DDP algorithm for solving essentially the exact problem we address here [5], but traditionally in robust control the particular formulation of uncertainty plays a big role in the quality (performance vs conservatism) of the solution. We are trying to continue their work by carefully investigating desirable formulations of terrain uncertainty and the best way to reflect them back into the limit cycle dynamics.

Approach

We consider a hybrid dynamical system where the only disturbance is the terrain height in the state transition set and the state transition function. For simplicity, we suppose there exists only one mode and one transition function.

$$\dot{x} = f(x,u) \quad \text{if} \quad g(x,u,h) < 0 \tag{1}$$

$$x^+ = \Delta(x^-,u,h) \quad \text{if} \quad g(x^-,u,h) = 0 \tag{2}$$

Where f is the continuous dynamics function, $g = 0$ represents the transition set, and Δ is the transition function.

Suppose the unknown terrain is bounded, we want to design a single trajectory $\Phi(x_0, u(.), t)$, such that it can hit all possible transition surfaces corresponding to heights within the given range.

Suppose for a terrain height h, the prior-impact state on the nominal trajectory is $x^-(h)$, with the corresponding post-impact state $x^+(h)$. We want to choose a meaningful cost function so that this open loop trajectory can reject terrain disturbance as much as possible. We continue to retain the constraint that the system has a perfect return map on the nominal flat terrain.

One way is to optimize the overall distance between the post-impact state $x^+(h)$ and the nominal trajectory $\Phi(x_0, u(.), t)$. We choose a phase variable to index the nominal trajectory, such as the stance leg angle of the walking robot. The distance between the post-impact state and the trajectory is computed as the weighed norm of the distance between $x^+(h)$ and the state on the nominal trajectory with the same phase.

Another candidate cost function is the gait sensitivity norm [3]. If a good indicator function g exists, we adopt the integral gait sensitivity norm $\int_{h^-}^{h^+} ||\partial g/\partial h|| dh$ as the cost function. Such norm can be computed using the shooting method, together with the norm's gradient.

We use SNOPT [2] to solve the NLP above. Due to sparsity of the gradient matrix, such NLP can be computed efficiently.

Open loop deadbeat controllers are computed using the scheme above for the SLIP model and the rimless wheel. For a more complicated system like the compass gait, the optimized trajectory can take twice as many as steps before falling than the passive limit cycle, i.e, no control input while walking down a shallow slope.

Discussion

- What is a good general cost function? The weighted norm approach appears to be a good candidate, but the weighted Euclidean distance does not capture the system dynamics.
- If the limit cycle computed from optimization is not self-stable, how can we construct a controller for stabilization?

Format

Poster

Keywords

legged locomotion, rough terrain, trajectory optimization

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