Indicators for Emergence of Double-limb Support in Passive Dynamic Walking

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1 Motivation and State of the Art

The effects of double-limb support (DLS) motion must play important roles in stable dynamic walking. The mechanism of DLS, however, has not been investigated in detail in the field of limit cycle walking because the stance-leg exchange is generally modeled on the assumption of inelastic collision; the rear leg leaves the ground just after the touchdown of the fore leg [1].

Based on the observations, the authors have investigated the potentiality of the emergence of DLS motion in passive cycle walking (PDW). We confirmed that a viscoelastic-legged rimless wheel (VRW) shown in Fig. 1 emerges the measurable period of DLS through numerical simulations and experiments [2]. The purposes of this study are to identify the conditions for the emergence of DLS motion in PDW and to specify the computational procedure for transition to DLS.

2 Our Approach

2.1 Viscoelastic-legged rimless wheel

Fig. 1 shows our experimental VRW (left) and the ideal model (right). This consists of eight identical telescopic-leg frames with viscoelasticity. Let c [N·s/m] be the viscosity coefficient and k [N/m] be the elastic coefficient. Let $\mathbf{q}^{\mathrm{T}} = \begin{bmatrix} x & z & \theta & L_1 & L_2 \end{bmatrix}$ be the generalized coordinate vector. The robot equation of motion then becomes

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}, \phi) = \mathbf{J}(\mathbf{q})^{\mathrm{T}}\boldsymbol{\lambda}, \quad \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}. \qquad (1)$$

The vector $\mathbf{h}$ includes the viscoelastic forces. The second equation represents the holonomic constraint condition. The Jacobian matrix, $\mathbf{J}(\mathbf{q})$, changes in accordance with the contact conditions. In the period of DLS, the following two conditions hold.

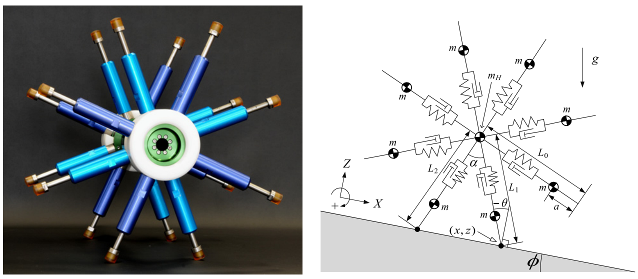

Figure 1: Experimental machine and its mathematical model of viscoelastic-legged rimless wheel

(C1) The end-point of Leg 1 (the fore support leg in the model of Fig. 1) contacts the floor without slipping.

(C2) The end-point of Leg 2 (the rear support leg in the model of Fig. 1) contacts the floor without slipping.

During DLS motion, we set $\mathbf{J}(\mathbf{q}) = \mathbf{J}_{\mathrm{DLS}}(\mathbf{q}) \in \mathbb{R}^{4\times5}$ that summarizes the constraint conditions of (C1) and (C2). Whereas during SLS motion, we set $\mathbf{J}(\mathbf{q}) = \mathbf{J}_{\mathrm{SLS}} \in \mathbb{R}^{2\times5}$ that summarizes the constraint conditions only of (C1). Let us divide the holonomic constraint force, $\mathbf{J}_{\mathrm{DLS}}(\mathbf{q})^{\mathrm{T}}\boldsymbol{\lambda}$ into

$$\begin{aligned} \mathbf{J}_{\mathrm{DLS}}(\mathbf{q})^{\mathrm{T}}\boldsymbol{\lambda} &= \begin{bmatrix} \mathbf{J}_1^{\mathrm{T}} & \mathbf{J}_2^{\mathrm{T}} & \mathbf{J}_3(\mathbf{q})^{\mathrm{T}} & \mathbf{J}_4(\mathbf{q})^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} \\ &= \mathbf{J}^{\mathrm{T}}\lambda_1 + \mathbf{J}^{\mathrm{T}}\lambda_2 + \mathbf{J}_3(\mathbf{q})^{\mathrm{T}}\lambda_3 + \mathbf{J}_4(\mathbf{q})^{\mathrm{T}}\lambda_4. \end{aligned} \qquad (2)$$

The Jacobian vectors $\mathbf{J}_1 \in \mathbb{R}^{1\times5}$ and $\mathbf{J}_2 \in \mathbb{R}^{1\times5}$ corresponds to the constraint condition (C1), whereas $\mathbf{J}_3(\mathbf{q}) \in \mathbb{R}^{1\times5}$ and $\mathbf{J}_4(\mathbf{q}) \in \mathbb{R}^{1\times5}$ corresponds to (C2). The vertical ground reaction force on the contact point of Leg 1 with the ground is λ_2, and that of Leg 2 is λ_4. We can detect the instant that Leg 2 leaves the ground by observing the sign of λ_4. The condition that both $\lambda_2 \geq 0$ and $\lambda_4 \geq 0$ hold is necessary for DLS.

2.2 Computational procedures based on DLS-Indicators

Let $\mathbf{f}$ be the vector of ground reaction force on the contact points of Leg 1 and Leg 2 at impact, and $\mathbf{J}_I(\mathbf{q})$ be the corresponding Jacobian matrix. The equation of motion then becomes

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}, \phi) = \mathbf{J}_I(\mathbf{q})^{\mathrm{T}}\mathbf{f}. \qquad (3)$$

Let $\mathbf{q}^{\dagger}$ and $\dot{\mathbf{q}}^{\dagger}$ be the vectors of angular position and angular velocity just before impact. We replace the position and velocity just before impact, $\mathbf{q}^{-}$ and $\dot{\mathbf{q}}^{-}$, with those exchanging Leg 1 and Leg 2 for the next ones, $\mathbf{q}^{\dagger}$ and $\dot{\mathbf{q}}^{\dagger}$[2].Note that $\mathbf{q}^{\dagger} = \mathbf{q}^{-} = \mathbf{q}^{+}$ holds where the superscripts "−" and "+" denote just before and just after impact. We assume that the touchdown of Leg 1 occurs at $t = T_0$ [s], that is, $\mathbf{q}^{\dagger} = \mathbf{q}(T_0)$. The time integral of the left-hand side of Eq. (3) then becomes

$$\begin{aligned} \lim_{\varepsilon\to+0} \int_{T_0-\frac{\varepsilon}{2}}^{T_0+\frac{\varepsilon}{2}} \left(\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}, \phi)\right) \mathrm{d}t &= \int_{T_0^-}^{T_0^+} \mathbf{M}(\mathbf{q}^{\dagger})\ddot{\mathbf{q}}\, \mathrm{d}t \\ &= \mathbf{M}(\mathbf{q}^{\dagger})\left(\dot{\mathbf{q}}^{+} - \dot{\mathbf{q}}^{\dagger}\right). \end{aligned}$$

The time integral of the right-hand side of Eq. (3) also becomes

$$\lim_{\varepsilon\to+0}\int_{T_0-\frac{\varepsilon}{2}}^{T_0+\frac{\varepsilon}{2}} \mathbf{J}_I(\mathbf{q})^{\mathrm{T}}\mathbf{f}\,\mathrm{d}t = \mathbf{J}_I(\mathbf{q}^{\dagger})^{\mathrm{T}} \lim_{\varepsilon\to+0}\int_{T_0-\frac{\varepsilon}{2}}^{T_0+\frac{\varepsilon}{2}} \mathbf{f}\,\mathrm{d}t = \mathbf{J}_I(\mathbf{q}^{\dagger})^{\mathrm{T}}\boldsymbol{\lambda}_I. \quad (4)$$

The collision equation then becomes

$$\mathbf{M}(\mathbf{q}^{\dagger})\dot{\mathbf{q}}^{+} = \mathbf{M}(\mathbf{q}^{\dagger})\dot{\mathbf{q}}^{\dagger} + \mathbf{J}_I(\mathbf{q}^{\dagger})^{\mathrm{T}}\boldsymbol{\lambda}_I, \;\; \mathbf{J}_I(\mathbf{q}^{\dagger})\dot{\mathbf{q}}^{+} = \mathbf{0}. \quad (5)$$

The Jacobian matrix at impact, $\mathbf{J}_I(\mathbf{q}^{\dagger})$, is exchanged in accordance with the following algorithm.

(A1) We set $\mathbf{J}_I(\mathbf{q}^{\dagger}) = \mathbf{J}_{\mathrm{DLS}}(\mathbf{q}^{\dagger})$ in Eq. (3) and compute $\boldsymbol{\lambda}_I \in \mathbb{R}^4$.

(A2) Divide $\boldsymbol{\lambda}_I$ into $\boldsymbol{\lambda}_I = \left[\,\lambda_{I1}\;\lambda_{I2}\;\lambda_{I3}\;\lambda_{I4}\,\right]^{\mathrm{T}}$. $\lambda_{I2} \geq 0$ and $\lambda_{I4} \geq 0$ must hold to transition to DLS. It is obvious, however, that $\lambda_{I2} > 0$ always holds. Therefore we should check the sign of λ_{I4} only.

(A3) If $\lambda_{I4} < 0$, DLS motion does not emerge. We then compute $\dot{\mathbf{q}}^{+}$ by setting $\mathbf{J}(\mathbf{q}^{\dagger}) = \mathbf{J}_{\mathrm{SLS}}$.

(A4) If $\lambda_{I4} \geq 0$, the motion then transitions to DLS and we compute $\dot{\mathbf{q}}^{+}$ by setting $\mathbf{J}(\mathbf{q}^{\dagger}) = \mathbf{J}_{\mathrm{DLS}}(\mathbf{q}^{\dagger})$.

Leg 2 begins to leave the ground at the instant that the sign of λ_4 in Eq. (2) continuously changes from positive to negative. The problem is that λ_4 just after impact, λ_4^{+}, does not always become positive even if λ_{I4} is positive in the case that the collision occurs while exerting the viscoelastic force. We then consider the following computational procedure.

(B1) If $\lambda_{I4} \geq 0$, the motion is determined to transition to DLS. The equations of motion just after impact are then specified as

$$\mathbf{M}(\mathbf{q}^{\dagger})\ddot{\mathbf{q}}^{+} + \mathbf{h}(\mathbf{q}^{\dagger},\dot{\mathbf{q}}^{+},\phi) = \mathbf{J}_{\mathrm{DLS}}(\mathbf{q}^{\dagger})^{\mathrm{T}}\boldsymbol{\lambda}^{+}, \quad (6)$$

$$\mathbf{J}_{\mathrm{DLS}}(\mathbf{q}^{\dagger})\dot{\mathbf{q}}^{+} = \mathbf{0}_{4\times 1}. \quad (7)$$

We then solve the equations for $\boldsymbol{\lambda}^{+} \in \mathbb{R}^4$ and $\ddot{\mathbf{q}}^{+}$.

(B2) Divide $\boldsymbol{\lambda}^{+}$ into $\boldsymbol{\lambda}^{+} = \left[\,\lambda_1^{+}\;\lambda_2^{+}\;\lambda_3^{+}\;\lambda_4^{+}\,\right]^{\mathrm{T}}$. If $\lambda_2^{+} \geq 0$ and $\lambda_4^{+} \geq 0$, then we take $\ddot{\mathbf{q}}^{+}$ obtained in (B1) as the proper initial acceleration vector and continue the numerical integral. The motion transitions to DLS.

(B3) If $\lambda_2^{+} \geq 0$ and $\lambda_4^{+} < 0$, unilateral constraint condition is not satisfied and the motion should transition to SLS. We then break $\boldsymbol{\lambda}^{+}$ and $\ddot{\mathbf{q}}^{+}$ obtained in (B1), and solve the following equations

$$\mathbf{M}(\mathbf{q}^{\dagger})\ddot{\mathbf{q}}^{+} + \mathbf{h}(\mathbf{q}^{\dagger},\dot{\mathbf{q}}^{+},\phi) = \mathbf{J}_{\mathrm{SLS}}^{\mathrm{T}}\boldsymbol{\lambda}^{+}, \quad (8)$$

$$\mathbf{J}_{\mathrm{SLS}}\dot{\mathbf{q}}^{+} = \mathbf{0}_{2\times 1}, \quad (9)$$

for $\boldsymbol{\lambda}^{+} \in \mathbb{R}^2$ and $\ddot{\mathbf{q}}^{+}$. We take these newly-calculated vectors as the proper initial conditions, and begin the numerical integral.

The measurable period of DLS emerges just after impact if λ_{I4}, λ_2^{+}, and λ_4^{+} are positive. These three quantities are thus termed as the DLS-Indicators (DLSIs).

3 Analysis Results

Fig. 2 plots the DLSIs with respect to the leg viscosity, c, in PDW on the slope of $\phi = 0.10$ [rad]. The physical parameters were chosen as listed in Table 1. See [2] for the details of the notations. Here, (a) plots λ_{I4} and (b) plots λ_2^{+} and λ_4^{+}. We also plotted their values with different symbols in the case that scuffing of Leg 2 arises; the rear leg quickly extends after takeoff where c is small and the end-point hits the ground. As seen from Fig. 2 (b), λ_4^{+} monotonically decreases as c increases and finally reaches zero where $c = 220$ [N·s/m]. This implies that unilateral constraint of Leg 2 just after impact does not hold and the stance-leg exchange is then completed instantaneously where $c \geq 220$. λ_{I4} is, however, always positive as shown in Fig. 2 (a). The Jacobian matrix in Eq. (5) is therefore chosen as $\mathbf{J}_I(\mathbf{q}^{\dagger}) = \mathbf{J}_{\mathrm{DLS}}(\mathbf{q}^{\dagger})$ and the impact model is different from the traditional one [1]. On the other hand, λ_2^{+} monotonically decreases as c decreases and finally reaches zero where $c = 4.0$ [N·s/m]. This implies that bouncing of Leg 1 arises where $c \leq 4.0$.

4 Open Questions

In this paper, we investigated the condition for emergence of measurable period of DLS motion and the corresponding computational procedure. It was clarified that DLSIs are useful for determining the motion subsequent to Leg 1's landing. In the case of λ_{I4}^{+} and $\lambda_4^{+} < 0$, however, unrealistic and complicated gaits are generated because Leg 2 hits the ground just after takeoff. The problem of how to deal with this case is left as a future work.

References

[1] T. McGeer, "Passive dynamic walking," *Int. J. of Robotics Research*, Vol. 9, No. 2, pp. 62–82, 1990.

[2] F. Asano and J. Kawamoto, "Passive dynamic walking of viscoelastic-legged rimless wheel," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, 2012. (to appear)

Table 1: Physical parameter settings

m_H	10.0	kg	L_0	1.0	m
m	1.0	kg	L^*	1.02	m
α	$\pi/4$	rad	k	500	N/m
a	0.3	m	ϕ	0.10	rad

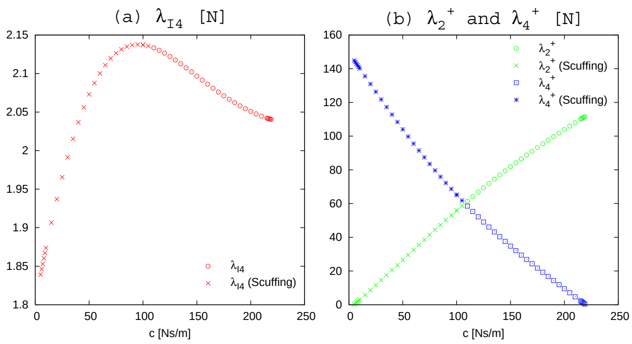

Figure 2: DLSIs with respect to c